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Photonic dispersion relation in a one-dimensional quasicrystal

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The dispersion relation of photons transmitting through a photonic one-dimensional quasicrystal arranged in a Fibonacci sequence was observed by measuring the spectrum of the phase change of the transmitted light using a Michelson-type interferometer. The phase spectrum obtained clearly showed the self-similar structure characteristic to dispersion curves of Fibonacci lattices.

Since the discovery of an Al-Mn alloy with icosahedral symmetry,[1] much attention has been focused on quasiperiodic systems. A lot of work has been concerned with the propagation of electrons or other classical waves in one-dimensional quasicrystals. Among them, Fibonacci lattices are of particular interest.[2–4] In Fibonacci lattices, all the states are critically localized. The energy spectrum has a fractal structure and forms a Cantor set with zero Lesbegue measure. Wave functions can also have a fractal behavior. Merlin *et al.*[5] have grown a Fibonacci lattice made of GaAs and AlAs and have studied x-ray diffraction and Raman scattering of it. Since then, several experimental studies have been reported on the wave propagation in Fibonacci lattices. Transmission of bulk acoustic phonons,[6] surface acoustic wave,[7] and third sound along superfluid helium thin film[8] have been studied. They gave the energy spectrum of these waves in the Fibonaaci lattices. The phase, however, of the transmitted waves was not measured in these studies. Dispersion relation of waves propagating in the quasiperiodic system can be obtained by measuring the amplitude and the phase of the transmitted wave.

In this paper, we report on our experimental study of the dispersion relation of photons propagating through a Fibonacci lattice composed of two dielectric materials. Photonic band formation in periodic dielectric structures and localization of light in random structures have been the subject of great interest in recent years.[9] Studies on photonic quasiperiodical structures will further extend this area of research. The photonic Fibonacci lattice was proposed by Kohmoto, Sutherland, and Iguchi.[10] They predicted a fractal behavior of the transmission spectrum. Compared with electronic systems, the advantage of studying photonic lattices is obvious. (i) Inelastic scattering of photons in the lattice can be negligibly small. (ii) There is essentially no interaction between photons. (iii) The reflectivity by a single interface can be relatively large. It is 23% in amplitude in the present study. It leads to the study of wave propagation in the regime where the single scattering approximation is not valid. (iv) Various sophisticated measurement techniques can be applied. Energy-resolved or time-resolved measurements with very high resolution are possible. The present study is a good example of this advantage.

A photonic Fibonacci lattice was prepared according to the model proposed by Kohmoto, Sutherland, and Iguchi.[10] The sample is a multilayer dielectric film of SiO_2 and TiO_2 deposited on a 1.5 mm-thick glass substrate. The sequence of the layers is based on the Fibonacci sequence, which is defined by

$$S_{j+1}=\{S_{j-1},S_j\},\quad S_0=\{A\},S_1=\{B\}. \tag{1}$$

In the present study the sequence was S_9. The number of the layers in S_9 is 55, which is Fibonacci number F_9. Layer A corresponds to SiO_2, and B to TiO_2. The optical thickness of each layer was designed to be 633 nm/4, i.e., $4n_Ad_A=4n_Bd_B=633$ nm, where n_A and n_B are the refractive indices of SiO_2 and TiO_2, and d_A and d_B are the physical thicknesses of layers A and B. These layers were deposited on a glass substrate by vacuum deposition, and the thickness of each layer was monitored by the change in the resonance frequency of a quartz oscillator. The calibration of the quartz oscillator was performed by measuring the wavelength of the maximum reflectivity for several thicknesses of single-layer films on a substrate. Since the reflectivity of a single-layer film is maximum or minimum at the quarter-wave condition, the resonance frequency change of the quartz oscillator for the quarter-wave thickness of each material for 633 nm was obtained by interpolating between the measured values.

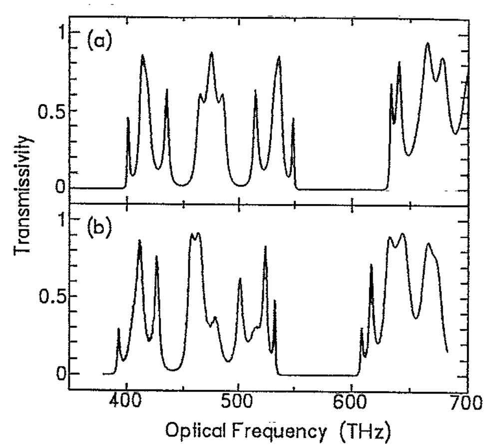

FIG. 1. The transmission spectrum of the sample obtained by (a) calculation based on an ideal Fibonacci lattice and (b) using a monochromator.

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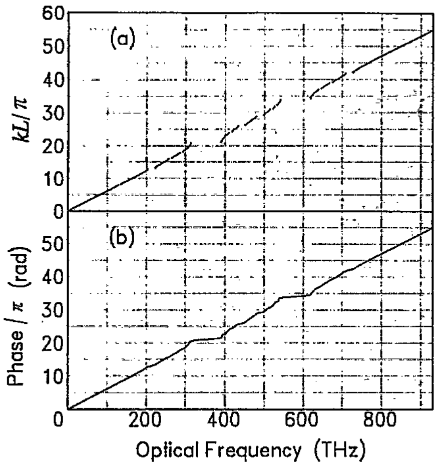

FIG. 2. (a) The calculated photonic dispersion relation of the periodic crystal which consists of the S_9 Fibonacci sequence. (b) The calculated phase of the light transmitted through the S_9 Fibonacci lattice from the air to the glass substrate.

The transmission spectrum of the sample is shown in Fig 1. Figure 1(a) is the calculated transmission spectrum of the Fibonacci lattice on a glass substrate. Interference of light due to the reflection by the other surface of the substrate was neglected. The values of the refractive indices used are 1.46, 2.35, and 1.5 for SiO_2, TiO_2, and the glass substrate, respectively. Figure 1(b) is the transmission spectrum of our sample obtained with a monochromator. Comparing Figs. 1(a) and 1(b) with each other, we see that correspondence of peaks and dips between the calculated one and the measured one is fairly good, except for a slight shift of the calculated spectrum to the higher frequency. We can attribute this shift to a small error in the calibration of the optical thickness of each layer. The best agreement between the calculated and the experimental spectra was obtained by assuming a value of 644 nm/4 instead of 633 nm/4. We will adopt this value in the following discussion in this paper. Looking into details of features in the two spectra, there are two wide gaps around 570 THz and below 380 THz, where the transmissivity is almost zero. The region between the gaps is divided into three by two transmission dips. Each of these three is divided into three again by two shallower dips. In Fibonacci lattices with increasing generations, the trifurcation is repeated, resulting in a self-similar structure of the transmission spectrum. This trifurcation structure is clearly seen in the experimentally observed spectrum. Imperfect agreement of the experimental spectrum with the calculated one is probably due to small fluctuations in the thickness of each layer and to optical dispersion of light in the materials.

In Fig. 2(a), the dispersion relation of photons in an S_9 Fibonacci lattice is shown. Periodic boundary conditions were used in the calculation. The gaps are located equidistantly at wave vectors of integer times of π/L with L being the total thickness of the lattice. The two largest gaps at 21π and 34π correspond to the gaps observed in the transmission spectrum, and other small gaps to transmission dips. The numbers 34 and 21 are Fibonacci numbers F_8 and F_7. The region between these gaps has a length of $F_6=13$. This region is divided by smaller gaps at 26π and 29π. The difference between these numbers and 21 or 34 are again Fibonacci numbers $F_5=8$ and $F_4=5$, respectively. For higher generations of Fibonacci sequences, the trifurcation structure grows self-similarly, and the ratio F_{j+1}/F_j converges to the golden mean, $(1+\sqrt{5})/2$. These beautiful properties of gap positions in a Fibonacci lattice, however, can only be observed when the wave vector of photons is measured. Because of the large modulation of the dielectric constant, the dispersion relation in the present sample is far from linear although the underlying dispersion relation of photons in free space is linear, and the gaps are not located equidistantly in energy space.

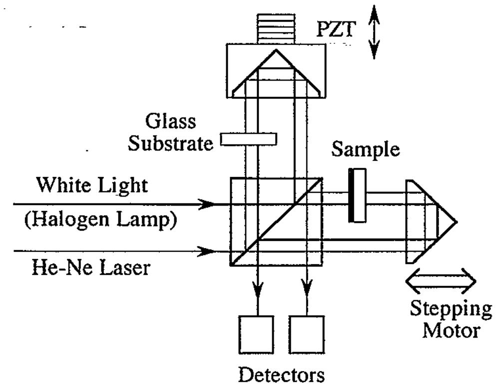

FIG. 3. The experimental schematic for the interferometric measurement of the amplitude and the phase of the light transmitted through a photonic Fibonacci lattice.

The wave vector of photons transmitting through a Fibonacci lattice can be obtained by measuring the phase of the light wave, since the phase, ϕ, is related to the wave vector, k, and the sample thickness, L, by $\phi=kL$. In Fig. 2(b) the calculated spectrum of the phase change of light transmitted through the S_9 Fibonacci lattice from the air side to the glass substrate is shown. Although discontinuities at gaps in the dispersion relation have disappeared, and the curve is smoothly connected because of the finite size of the system, the positions of the gaps and the overall shape are reproduced very well in the phase spectrum.

Experimentally, the amplitude and the phase of the light transmitted through the sample were obtained by a Fourier-transform analysis of interferograms obtained by a Michelson-type interferometer as shown in Fig. 3. The light source for the interferometer was a halogen lamp, and the light was focused to a 100-μm pinhole and collimated by a lens. The light was divided into two arms by a beam splitter cube, reflected by corner-cube prisms, combined by the same beam splitter, and detected by a photomultiplier tube. The path length of one arm was modulated at 20 kHz by a piezoelectric actuator attached to the corner-cube prism. The amplitude of the path-length modulation was about 250 nm. The path length of the other arm could be scanned by a stepping-motor-driven translation stage (Suruga, R10-80L). One step of the stepping motor nominally corresponded to a path length of 44 nm. The output of the photomultiplier tube was fed into a lock-in amplifier, and the signal at the frequency of the path-length modulation was measured. A beam of light at 632.8 nm from a He-Ne laser was passed in the interferom-

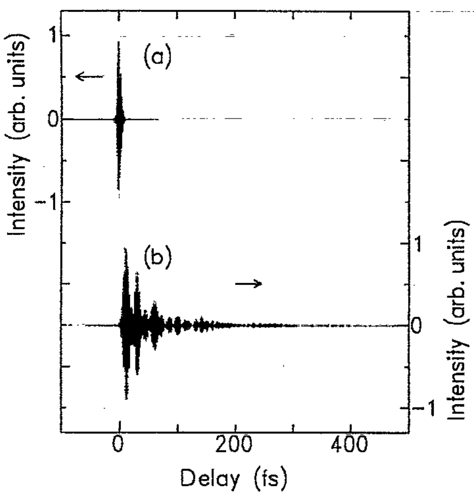

FIG. 4. The interferograms obtained by the Michelson-type interferometer. (a) The autocorrelation. (b) The cross correlation.

eter parallel to the white-light beam, and the interferogram was simultaneously monitored for the calibration of the path length.

Two interferograms were obtained using this setup. One was the autocorrelation of the white light. The other was the cross correlation of the light before and after transmisson through the Fibonacci lattice. The cross correlation was obtained by inserting the Fibonacci lattice on a glass substrate into one arm of the interferometer, and a glass substrate of the same thickness without a Fibonacci lattice on it into the other arm. Thus the change of the optical path length due to the substrate was compensated for. The complex transmission coefficient, $t(\omega)$, of the Fibonacci lattice can be obtained by the following equation:

$$t(\omega)=\frac{\mathscr{F}\{C_C(\tau)\}}{\mathscr{F}\{C_A(\tau)\}}=|t(\omega)|\exp[i\phi(\omega)]. \qquad (2)$$

Here, $C_A(\tau)$ and $C_C(\tau)$ are the autocorrelation and the cross correlation, with τ being the delay time, and $\mathscr{F}\{\ \}$ denotes the Fourier transform of the function in the bracket. The transmissivity of light through the sample is the squared magnitude, $|t(\omega)|^2$, and the phase spectrum of the light is given by $\phi(\omega)$.

The autocorrelation and the cross correlation obtained in this interferometric measurement are shown in Fig. 4. These are not the autocorrelation or the cross correlation of the light in a strict sense, since we measured the signal amplitude by the path-length modulation technique. The signals obtained correspond to the time derivative of the true correlation when the modulation amplitude is small compared with the wavelength of the light. The transmission coefficient, however, obtained by Eq. (2) is not affected by this fact. In this sense, we can safely treat them as the autocorrelation or cross-correlation functions. We note here that the obtained cross correlation can be regarded as the wave shape of the optical pulse after the transmission through the Fibonacci lattice which originally had a wave shape of the autocorrelation. The temporal width of the autocorrelation, which was about 6 fs, determines the time resolution of this time-domain measurement. This interferometric technique using white light for the time-domain observation of time-of-flight profiles with ultrahigh time resolution can be applicable to studies of various types of coherent propagation phenomena. The time-of-flight profile of the Fibonacci lattice is expected to simply represent the spatial density correlation of the lattice in the limit of small refractive-index modulation. It is not the case, however, with the present sample, where the refactive-index ratio is as large as 1.6. Thus a Fourier-transform analysis of the correlation data is required to elucidate the nature of the photonic state in the lattice.

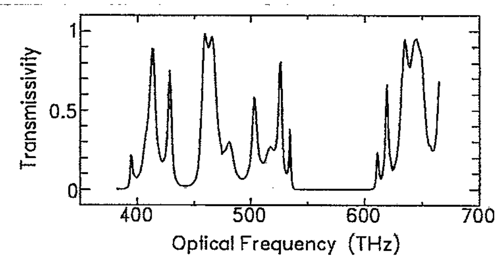

FIG. 5. The transmission spectrum of the sample obtained by Fourier transformation of the interferograms measured by using the Michelson interferometer.

The transmissivity obtained by the Fourier transformation is shown in Fig. 5. The remarkable agreement of it with the spectrum obtained by a conventional method using a monochromator demonstrates the accuracy of the present Fourier-transform technique. In Fig. 6, the phase spectrum of the transmitted light obtained by the Fourier transformation is shown. Shifts of the phase by multiples of 2π are arbitrary. Because of imperfect compensation of the path-length change due to the substrate, the resulting phase spectrum can have a small offset proportional to the frequency. By matching the spectrum obtained with the calculated one, the thickness of the glass substrate without a sample was estimated to be 2.9 μm larger than that of the substrate with the Fibonacci lattice on it. The spectrum shown is a corrected one using this value. At the frequency regions of the two largest gaps the phase is not plotted since the transmitted light was too weak there to obtain a value of the phase with enough accuracy. The phase spectrum obtained shows very good agreement with the calculated one as shown in Fig. 2(b), and

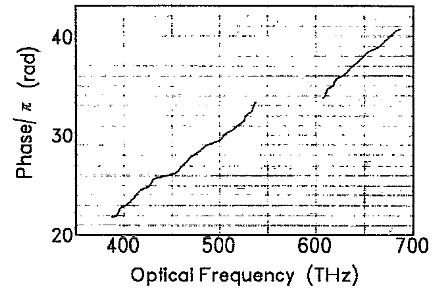

FIG. 6. The phase of the light transmitted through the Fibonacci lattice obtained by the Fourier-transform interferometery.

clearly shows the self-similar structure of the dispersion curve typical of Fibonacci lattices.[11] Besides the positions of the largest gaps, smaller gaps are also clearly observed at the right positions, and the overall shape of the spectrum agrees very well with the calculated phase spectrum. Thus we could actually observe the dispersion relation of photons in a one-dimensional quasiperiodic lattice experimentally.

In summary, a photonic Fibonacci lattice was made by depositing two dielectric materials on a glass substrate, and the dispersion relation of photons was experimentally observed by a Fourier-transform technique using a Michelson-type interferometer. The phase spectrum obtained clearly shows the self-similar structure of the dispersion curve typical of Fibonacci lattices.

Recently, we have been informed that Gellermann *et al.*[12] have prepared similar Fibonacci samples, and observed their transmission spectra.

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[1] D. Shechtman, I. Blech, D. Gratias, and J. W. Cahn, Phys. Rev. Lett. **53**, 1951 (1984).

[2] M. Kohmoto, L. P. Kadanoff, and C. Tang, Phys. Rev. Lett. **50**, 1870 (1983).

[3] S. Ostlund, R. Pandit, D. Rand, H. J. Schellnhuber, and E. D. Siggia, Phys. Rev. Lett. **50**, 1873 (1983).

[4] M. Kohmoto, B. Sutherland, and C. Tao, Phys. Rev. B **35**, 1020 (1987).

[5] R. Merlin, K. Bajema, R. Clarke, F.-T. Juang, and P. K. Bhattacharya, Phys. Rev. Lett. **55**, 1768 (1985).

[6] D. C. Hurley, S. Tamura, J. P. Wolfe, K. Ploog, and J. Nagle, Phys. Rev. B **37**, 8829 (1988).

[7] L. Macon, J. P. Desideri, and D. Sornette, Phys. Rev. B **44**, 6755 (1991).

[8] D. T. Smith, C. P. Lorenson, and R. B. Hallock, Phys. Rev. B **40**, 6634 (1989).

[9] *Photonic Band Gaps and Localization*, edited by C. M. Soukoulis (Plenum, New York, 1993).

[10] M. Kohmoto, B. Sutherland, and K. Iguchi, Phys. Rev. Lett. **58**, 2436 (1987).

[11] F. Nori and J. P. Rodriguez, Phys. Rev. B **34**, 2207 (1986).

[12] W. Gellermann, M. Kohmoto, B. Sutherland, and P. C. Taylor, Phys. Rev. Lett. **72**, 633 (1994).